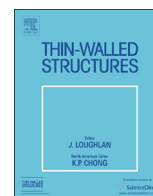




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Review

Liquid-containment shells of revolution: A review of recent studies on strength, stability and dynamics



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ABSTRACT

In civil engineering, shell structures are widely used as liquid-containment vessels. Understanding how the shell responds to relevant loading conditions is important for the design of safe and economical liquid-containment shell structures. This paper reviews recent research on the strength, stability and vibration behaviour of liquid-containment shell structures, and traces the developments pertaining to the design of these facilities to withstand various loading and environmental effects such as liquid pressure, wind pressure, ground movement and thermal effects. Results of recent feasibility studies of non-conventional shell forms for liquid containment are also reported, and areas of focus for future research are suggested.

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1. Introduction

Shells are widely used in the civil engineering industry for liquid containment [1,2]. Applications include elevated water tanks, storage vessels for the containment of petroleum products, liquefied gases and industrial chemicals, and water-treatment structures such as settling tanks and sludge digesters.

The vast majority of industrial metal tanks are of cylindrical (and to a lesser extent conical) shape owing to the ease of fabrication of shells of single curvature. Metal tanks of double curvature include spherical, ellipsoidal and toroidal vessels. For elevated water reservoirs, the mouldability of concrete into any desired shape has allowed these tanks to be constructed in a great variety of interesting shapes. In the 50 years or so, the same versatility of concrete gave rise to many architectural forms in the area of shell roof construction.

Thin shells have the advantages of high strength-to-weight ratio, functional effectiveness (excellent shape for containment), and good aesthetics. However, the property of thinness attracts special problems. One of the challenges is predicting how the shell

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responds to extreme loading and environmental conditions, which include liquid pressure (hydrostatic and hydrodynamic), wind pressure, sudden ground movements (earthquakes), impact, blast and temperature gradients. This understanding is vital for the design of safe and economical liquid-containment shell structures satisfying key performance requirements.

Over the past five decades, fundamental research on mathematical theories of shells has gradually given way to computational formulations such as the finite element method (FEM), the boundary element method (BEM) and the finite difference method (FDM), which are well suited to the study of complex shell problems. These methods have formed the basis of many studies of shell structures. Experimental methods have also played an important role, not only in their own right, but also as validation of the generally more economical numerical methods.

In particular, containment shell structures have been the subject of intense research over the last 50 years, and the literature in this area is substantial. Many review articles covering various aspects of shell structures have also appeared in the past. As far back as 1982, Tooth [3] surveyed storage vessels as an application of shells, while in 1996, Teng [4] reviewed the field of shell buckling, including tanks and silos.

This paper is based on a lecture that was presented at an international conference in 2012 [5]. We survey developments that have been reported in the literature since the turn of the millenium, with regard to a better understanding of the strength, stability and vibration behaviour of liquid-containment shell structures. These efforts have generally aimed at quantifying the relevant effects (critical buckling loads, natural frequencies, governing stresses, maximum deformations, etc), and proposing suitable design recommendations. It must be pointed out that there are many papers on the subject that have been published prior to 2000, but since the intention is to capture current trends in the field, it has been considered necessary to go back only as far as 2000; this period features a sufficiently large volume of literature to allow us to see the most significant trends. Reviews covering earlier contributions may be seen elsewhere in the literature [3,4].

Only shells of homogeneous construction are covered; laminated, sandwich and composite shells are outside the scope of this survey. Boilers and pressure vessels are also not included. The considerations in the earlier part of the paper mainly relate to metal shells, but studies reported in the later part of the paper are more relevant to construction in concrete. The aim of this review is not to look at every paper on liquid-containment shells that has appeared in the literature since 2000 (they are too many), but rather, to discuss the more representative of these studies, thus showing the recent areas of focus and general trends in research. Aspects of recent work of the author on the feasibility of new shell

forms for liquid containment are also discussed. At the end, areas of focus for future research are suggested.

2. Buckling of vertical cylindrical tanks

Vertical cylindrical shells offer a convenient solution for the storage of water, petroleum products or chemicals, on account of the ease of manufacture of the cylindrical form (with its single curvature), the good containment properties of the cylindrical shape, and the structural efficiency of an axisymmetric distribution of primary loading (hydrostatic pressure). Problems associated with the buckling of the shell have been studied the most, given that the wall thickness t of these tanks is generally very small in relation to the radius r of the tanks. Metal tanks, where the r/t ratio typically lies in the range $500 \leq r/t \leq 2000$, are particularly vulnerable to buckling instability.

The buckling strength of a cylindrical shell subjected to an axial compressive load is a problem that has been studied many times, and interest on this topic still continues. With the development of design guidelines in mind, Kim and Kim found, in a study published in 2002 [6], that the buckling strength of such shells decreased significantly as the r/t ratio increases, while buckling strength decreased only slightly as the h/r (height-to-radius) ratio increased. A regression analysis of the numerical results led them to the formula:

$$\frac{\sigma_{cr}}{E} = 1.19 \left(\frac{H}{D} \right)^{-0.0256} \frac{t}{D} \quad (1)$$

where σ_{cr} is the critical buckling stress, E the Young modulus, H/D the height-to-diameter ratio of the cylinder, and t/D the thickness-to-diameter ratio. For a thin steel shell with an H/D ratio of 1.0 and $t/D = 1/1000$, this would give a σ_{cr} value of 238 N/mm^2 (assuming an E value for steel of $200 \times 10^9 \text{ N/m}^2$), which seems a little too high. Their formulation assumed that the cylinder is geometrically perfect.

The more general problem of the stability of circular cylindrical steel shells under simultaneous axial compression, torsion and external pressurisation (Fig. 1) was studied by Winterstetter and Schmidt [7]. In a paper published in 2002, they presented a proposal for the interactive buckling design of cylindrical steel shells on the basis of a comprehensive set of experimental and numerical results, for various types of analysis ranging from linear to fully nonlinear, as defined in Eurocode 3 Part 1–6 [8].

Cylindrical steel tanks with stepwise-varied wall thickness are a common form of construction. Membrane hoop stresses in the shell due to hydrostatic loading increase linearly with depth below the surface of the liquid, so it is reasonable to increment the thickness of the tank wall as one moves along the shell meridian

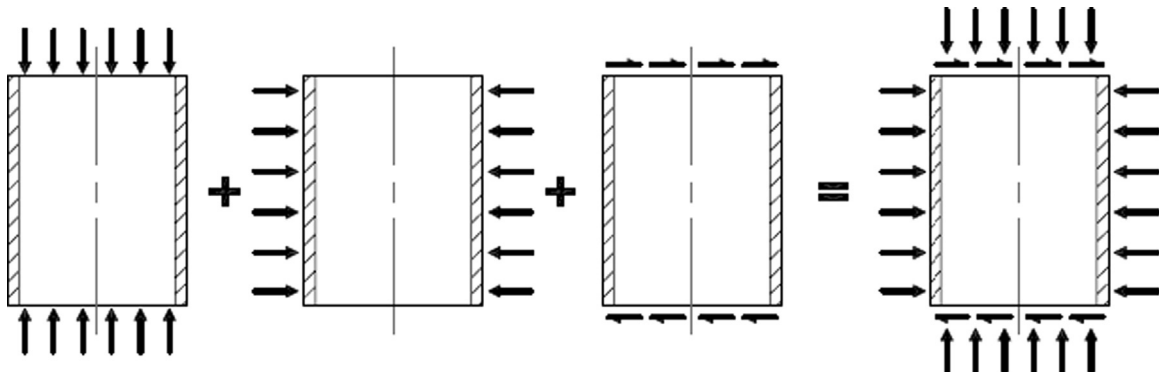


Fig. 1. Cylindrical shell under axial compression, external pressure, torsion and combined loading [7].

from top to bottom. For practical reasons, the thickness enhancement is effected at a finite number of locations, by attachment of additional wall plates. Chen and co-workers [9] have investigated the buckling of stepwise-thickened cylindrical shells under uniform external pressure. They have proposed a simplified method utilising the concept of a “weighted smeared wall”, suitable for hand calculations. Their approach appears to give good estimates of buckling strength for short to medium-length shells.

The rather different problem of metal tanks supported on columns and other discrete supports has been considered by Guggenberger and his collaborators [10]. This problem is characterised by stress concentrations in the vicinity of the discrete supports, and bending-related local effects. They focussed attention on the buckling strength of the shell in the regions above the local supports, assuming that there is no provision of a ring beam or shell stiffeners in the region of the supports. Using finite element modelling, they investigated the variation of the elastic buckling resistance with support width for the full range of possible support widths. Although the results were based on a single r/t ratio of 500 and a single h/r (height-to-radius) ratio of 2.0, valuable insights were gained.

Wind pressure on cylindrical tanks represents a rather complex loading situation. Over the past 10 years, a considerable amount of attention has been devoted to investigating this problem. Internal hydrostatic pressure of the liquid tends to stabilise the walls of large storage tanks. Such tanks are most vulnerable to wind-induced buckling when they are empty. Portela and Godoy investigated cylindrical tanks surmounted by a conical closure in one study [11], and tanks with a dome-shaped roof in another [12]. Wind-tunnel experiments were first carried out to determine wind pressures, and the results used as loading data in finite-element computational models of the tanks. They found that buckling was mainly confined to the cylindrical part of the tank on the windward side. It seemed that buckling was induced by local effects due to positive wind pressure on the windward side, and hardly affected by the negative pressure distribution around the tank. As expected, the provision of a roof was found to stiffen the structure as a whole. Thus the tank with a conical roof had a larger buckling resistance than a similar tank without a roof.

The critical load obtained by a normal bifurcation analysis is, of course, only an upper bound to the actual buckling loads in the real structure, and hence is not safe for design purposes. On the other hand, a fully nonlinear incremental analysis accounting for imperfections can be computationally expensive, and unjustifiable in the early stages of design. To get around this problem, Jaca et al. [13] used the concept of a “reduced stiffness” in a simple eigenvalue buckling analysis of open cylindrical tanks under wind loads, in an attempt to obtain a lower-bound estimate of shell buckling loads. The essence of this approach was the neglecting of the membrane contribution to the stiffness matrix. In a follow-up paper, Sosa and Godoy [14] developed an implementation of this lower-bound approach for the buckling of imperfection-sensitive shells. They considered cylindrical tanks with conical roofs and with open tops, and arrived at some knock-down factors for various values of H/D ratios of the tank. For tanks with conical roofs, the lower-bound values predicted by the reduced energy method was found to be unsafe for design, since the critical load given by a more accurate nonlinear analysis (accounting for imperfections) was 10% lower.

Very recently, Zhao and Lin [15] have used the finite element method to investigate the buckling behaviour of open-top cylindrical steel tanks under wind load, for tanks whose H/D ratio does not exceed 1.0. They have found that the buckling behaviour of tanks subjected to wind load is primarily governed by windward positive pressure, and hardly influenced by pressures elsewhere on the tank. These findings are similar to the 2005 observations of

Portela and Godoy with regard to closed-top tanks [11,12]. Uematsua and co-workers [16] have also very recently used wind tunnel measurements and finite element analysis to derive wind pressure coefficients for designing open-top oil storage tanks. They too have established that the buckling behaviour of the tank is mainly governed by the magnitude and distribution of the positive wind-pressure coefficients on the windward surface of the tank. The development of design-oriented wind-pressure coefficients for tanks is a useful addition to the literature.

As is well known, the response to wind of a structure (whether this be a building, an industrial tower or a tank) is very much influenced by the presence of other structures in the vicinity. Of particular interest to researchers has been the nature of wind loads within a group of tanks. Burgos et al. [17] have focussed on the interaction of two tanks. They have conducted wind tunnel experiments to determine the pattern of pressure distribution on a tank which is shielded by another tank, and then used this data as input for a finite-element buckling analysis of the tank. They have noted a reduction of up to 30% in buckling capacity of the tank as a direct result of the interference effects between the two tanks.

In a recent study, Zhao and co-workers [18] have conducted a large number of wind tunnel tests to establish the wind loads on large open-top cylindrical tanks. Taking the case of the wind load on an isolated tank as a benchmark, they have considered various configurations of tanks, and determined the wind loads for each. The main contribution of this work is the provision of more specific wind data for particular arrangements of tanks. It extends the work of Burgos et al. [17] to groups of more than two tanks. However, an extension to yet other possible group configurations is necessary, to afford a more comprehensive set of guidelines for practical design. Thus, there is scope for more research here.

Cylindrical tanks containing flammable products are vulnerable to failure by thermal buckling in the event of fire. Prompted by the frequent occurrence of fire incidents at petroleum depots in South America, Godoy and Batista-Abreu [19] numerically investigated fire-induced buckling of oil-storage cylindrical steel tanks, and performed a parametric study on the influence of shell thickness, level of oil in the tank, area in contact with the fire, and other factors. They found that the temperatures required to induce elastic buckling can be as low as 100–200 °C. The actual values largely depend on the temperature gradient across the shell thickness. The response of the shell is complex, as it depends on so many parameters. It is clear that a lot of work is still required in order to fully understand the buckling response of oil-filled cylindrical tanks under fire conditions.

As is well known, the buckling strength of thin metal shells is considerably less than the predictions of theory, owing to the existence of imperfections in the shell, typically geometric imperfections and material imperfections. The influence of imperfections on the buckling behaviour of cylindrical tanks and silos is a subject that has continued to receive the attention of researchers worldwide. Pircher and Bridge [20] studied the buckling of tanks and silos with circumferential weld-induced imperfections. They found that weld-induced residual stresses had a small beneficial effect (the buckling load increased slightly), while interaction between neighbouring circles of imperfections was found to reduce the buckling strength.

The shape of the imperfection induced by welding has an influence on the buckling resistance of the shell, and so not surprisingly, many investigators have come up with various models which attempt to describe the shape. Pircher et al. [21] have proposed a shape function to describe the geometry of a circumferential weld imperfection, by making use of elastic shell theory (axisymmetric bending of cylindrical shells), in combination with imperfection measurements in real shells. The transverse displacement of the shell midsurface caused by the weld, denoted

by $w(x)$, was approximated as

$$w(x) = w_0 e^{-nx/\lambda} \left(\cos \frac{\pi x}{\lambda} + \zeta \sin \frac{\pi x}{\lambda} \right) \quad (2a)$$

where

$$\lambda = \pi \sqrt{\frac{rt}{\sqrt{12(1-\nu^2)}}} \approx 2.444\sqrt{rt} \quad (2b)$$

and ζ ($0 \leq \zeta \leq 1$) is a weld-stiffness parameter ($\zeta = 1$ denotes full moment continuity, while $\zeta = 0$ denotes a hinge), x is the distance coordinate from the weld, and w_0 is the imperfection amplitude (the maximum value of w occurring at $x=0$). Curve fitting was then performed using measured data, to determine the parameters of the shape function.

Large tanks are often assembled by welding together a number of cylindrical panels. In a study published in 2002, Hornung and Saal [22] performed buckling tests on four such tanks with an internal vacuum pressure. In the tests, geometric imperfections around the welds were measured at various locations, and used as input in the analysis. The measurements showed that the sizes of geometric imperfections in real tanks often exceed the tolerance criteria in design standards for stability of shells, such as the German standard (DIN) and the Eurocode standard. When observed buckling loads were compared with predictions of design standards, significant discrepancies were noted. The experimental results of Hornung and Saal indicated a need for a review of current guidelines. Four years later, Hubner et al. [23] proposed a simple numerical method for the simulation of weld depressions and associated residual stresses, and conducted finite-element buckling analyses based on the model. This model gave buckling loads that were in close agreement with the predictions of the Eurocode standard, EN1993 Parts 1–6 [8].

Although good numerical modelling can now simulate weld-related imperfections with reasonable accuracy, experimental testing (small-scale or full-scale) remains indispensable in seeking a better understanding of the buckling behaviour of imperfect cylindrical steel shells. Teng and Lin [24] have proposed the use of small-scale laboratory models. They have presented a procedure for the fabrication of such models, for a more realistic simulation of the effect of welds in large tanks and silos.

Foundation behaviour has a strong bearing not only on the overall stability of a cylindrical tank, but also on the resistance of the thin shell to local buckling. A number of investigations in the past 12 years have focussed on the effects of support settlement on the stability and buckling resistance of cylindrical tanks. In a paper published in 2003, Godoy and Sosa [25] studied the problem of differential settlement of storage tanks, and using FEM modelling, investigated how the central angle of the zone affected by settlement (i.e. the subtended angle), and the amplitude of the settlement, affected the magnitude of the out-of-plane displacements that triggered buckling. In cases where the differential settlement around the edge of a large oil tank can be approximated as a harmonic distribution, Gong et al. [26] have found that the number of waves of settlement along the circumference has a considerable influence on the buckling behaviour of the tank.

Mark et al. [27] considered the effect of local differential settlement on the buckling behaviour of a thin cylindrical shell under axial compression. Such local settlements induce imperfections in the shell, which in turn then affect the buckling behaviour. They observed that a relatively small local uplift displacement at the shell boundary can cause a snap-through buckle that forms a dimple in the shell, and with the further imposition of uniform axial compression, the dimple grows and migrates; buckling of the whole shell then occurs by propagation from the dimple.

A comprehensive guide to the design of metal shells, intended to complement Eurocode 3 Part 1.6 [8], has been compiled by the European Convention for Constructional Steelwork [28]. This mainly covers the buckling resistance of cylindrical and conical shells, but also includes information on spherical and torispherical shells under uniform pressure.

3. Dynamic response of vertical cylindrical tanks

Vertical cylindrical tanks are also vulnerable to dynamic effects arising from earthquakes and wind. Large storage tanks are particularly vulnerable to the effects of sudden ground movements generating inertial forces.

The nonlinear analysis of liquid-storage tanks under earthquake excitation has been tackled by Wunderlich and Seiler [29]. They adopted a nonlinear finite-element procedure that used pressure eigenvectors as equivalent loads, thus avoiding solving the coupled problem in the time domain. This quasi-static approach yielded some useful insights on the behaviour of the tanks, including phenomena like “elephant footing” and buckling in the uppermost regions of the shell. The study (reported in 2000) also revealed some shortcomings in Eurocode 8 Part 4 (Seismic Design of Tanks, Silos and Pipelines) [30].

Three years later, Nachtigall et al. [31] addressed essentially the same problem of the structural response of vertical cylindrical tanks under earthquake excitation, and obtained results which appeared to be an improvement over those predicted by Eurocode 8 Part 4 [30], and by the American Petroleum Industry (API) Standard 650 (Seismic Design of Storage Tanks) [32]. In their approach, these investigators discarded the simplification of the cylindrical tank as a vertical cantilever beam (as adopted by some previous investigators), and instead used the shell’s mode shapes (when empty) as a basis for approximating the coupled behaviour of the tank-liquid system.

Fundamental modes of vertical cylindrical tank-liquid systems have been investigated by Virella and co-workers [33]. Based on finite-element modelling, they obtained natural frequencies, mode shapes and dynamic response of anchored tanks under horizontal ground motion. Their investigation centred on tanks with three H/D ratios: 0.95, 0.63 and 0.40. The mass of the liquid in the cylinder was lumped at the nodes of the cylinder in accordance with a defined law. Clearly this is an approximate approach, but the results obtained agreed well with those of a more exact FEM model in which the liquid was represented by acoustic finite elements.

Virella and his co-investigators found that the response of a tank-liquid system subjected to a horizontal ground motion can be accurately predicted by considering just the fundamental mode, which is a bending mode, regardless of the H/D ratio. For practical ranges of shell dimensions encountered in the oil industry, the fundamental modes are therefore not those associated with circumferential wave patterns, a finding that seems to be in contradiction to the earlier work of Nachtigall et al. [31]. In a follow-up paper [34], the authors considered the dynamic buckling of the same steel shells subjected to real earthquake records. The buckling modes were found to exhibit significant deflections at the top of the cylindrical tank. The authors attributed this behaviour to the occurrence of a negative (inward) net pressure in the zone where the impulsive hydrodynamic pressure generated by the earthquake excitation exceeded the hydrostatic pressure, inducing membrane compressive stresses that triggered buckling of the shell.

In order to fully understand the response of a storage tank under severe earthquakes, where other degrees of freedom may arise, Taniguchi [35] studied the rocking motion of un-anchored

cylindrical tanks with flat bottoms. Ahari et al. [36] performed an uplift analysis of the bottom plate of un-anchored cylindrical steel storage tanks, where the plate was modelled as a tapered beam resting on a rigid foundation. Their results did not differ significantly from those of earlier investigators who had used constant-width beam models. Taniguchi et al. [37] presented a procedure for evaluating the liquid pressure on the rigid flat bottom of a cylindrical tank, caused by the uplift motion of the un-anchored base. More recently, Ozdemir et al. [38] employed a fully nonlinear fluid–structure interaction FEM modelling for the seismic analysis of both anchored and un-anchored tanks. They validated their modelling through comparisons with available experimental results, and went on to evaluate the provision for seismic design as contained in various seismic standards (API 650, Eurocode 8 and others).

Relatively recently, Amiri and Sabbagh-Yazdi [39] have investigated the influence of the tank roof on the dynamic characteristics of tanks, by comparing the results for tanks with roofs versus those for tanks without roofs. Assuming the bottoms of the tanks were anchored to the foundation, they studied tanks with H/D ratios of 1.0, 0.76 and 0.67, for various liquid levels in the tank. The main finding was that the influence of the roof on the natural frequencies of axial modes is negligible, whereas its influence on the natural frequencies of circumferential modes is significant.

To safeguard liquid-storage tanks against excessive earthquake damage, strengthening may be adopted, but this may not always be effective. Alternatively, the tanks may be provided with devices which decouple the structure from the ground, in order to reduce the peak response of the structure. This strategy is called “base-isolation”, and a number of studies have been reported on the dynamic response of cylindrical storage tanks which are base-isolated. Kim and co-workers [40] employed a hybrid formulation, where finite elements were used to model the shell, and boundary elements were used to model the liquid and the soil. The structural problem thus comprised three sub-systems: a liquid–structure interaction system (superstructure), a soil–structure interaction system (substructure), and a base-isolation system (in-between). This is depicted in Fig. 2.

The main finding of Kim and co-workers [40] was that the relative displacements between the tank and the foundation have their greatest values at the lower frequencies of excitation, and where low-frequency earthquakes occur, base-isolation is very effective in reducing the seismic force transmitted from the ground to the structure. More recently, Shekari et al. [41] also employed a hybrid formulation, where the shell domain was modelled by finite elements and the fluid domain was modelled using the boundary element method (BEM). Solving the fluid–structure interaction problem, they found that seismic isolation is more effective in slender tanks than in broad tanks, and flexible

isolators reduce the seismic response more effectively than stiff isolators. In a follow-up paper [42], considerations were extended to the particular circumstances of long-period ground motions.

The sloshing of liquid in storage tanks can result in undesirable dynamic effects on the shell. To suppress sloshing, baffles are usually installed in the tank. Baffle parameters (such as shape, location, spacing, etc) significantly influence the dynamic characteristics of the fluid–structure interaction problem, and a number of investigations has centred around baffle effects. Cho et al. [43] employed the structural-acoustic FEM formulation to investigate the free vibration response of storage tanks with horizontal annular baffles. They carried out a comprehensive parametric study of the influence of baffle number, baffle location, baffle inner-hole diameter, liquid level, etc, on the natural frequencies and mode shapes of cylindrical tanks.

Other than earthquakes, dynamic effects in vertical cylindrical tanks may also be induced by wind. Wind gusts generate transient vibrations in the shell, which may lead to dynamic buckling. Dynamic buckling occurs when relatively small oscillations under an excitation suddenly become much bigger in amplitude. In a paper published in 2005, Sosa and Godoy [44] considered steel tanks with a fixed conical roof, and concluded that dynamic effects do not significantly influence buckling behaviour for short tanks. For such tanks, static buckling models provide a reasonable approximation of the buckling strength of the shell under wind. Very recently, and adopting the added-mass technique to dynamically account for the presence of liquid in the tank, Buratti and Tavano [45] have investigated critical buckling loads of cylindrical steel tanks that are fully anchored at the base and experiencing earthquake-induced ground motions, and obtained seismic fragility curves for these.

4. Mechanics of horizontal cylindrical tanks

Horizontal cylindrical tanks find application in underground liquid storage, vehicle-mounted bulk-liquid transportation, and industrial liquid containment. The liquids contained in such tanks are usually fuel and other petroleum products, liquefied gases, industrial chemicals, and processed liquid foods like milk.

Fig. 3 shows ground-mounted and buried horizontal circular-cylindrical tanks supported at the ends. Such tanks have end closures typically in the form of circular flat plates, shallow spherical shells, ellipsoidal shells or tori-spherical shells. Supports may also be provided in the form of saddles positioned at two or more locations along the length of the vessel.

The buckling of horizontal cylindrical vessels supported on two saddles were investigated by Chan et al. [46]. For the loading patterns that were considered, the buckling behaviour of the vessel was primarily governed by the values of longitudinal and circumferential membrane stresses at the top and bottom locations of the midspan section and the sections through the centre of bearing of the saddles. By comparing allowable buckling stresses derived from known buckling formulae with stresses calculated from a linear elastic analysis of the shell, they proposed a simplified design method for saddle-supported cylindrical vessels. Based on the results of a parametric study, Banks et al. [47] presented a simplified method for determining the maximum strain in a plastic cylindrical tank supported on two saddles.

In a study published in 2003, Magnucki and Stasiewicz [48] considered the stability of horizontal cylindrical tanks with ellipsoidal closures. Ground-mounted tanks were assumed to be primarily loaded by internal hydrostatic pressure, while buried tanks were assumed to be empty and subjected to external hydrostatic pressure from the water in the soil. The approach of Magnucki and Stasiewicz was based on Donnell’s equation for the

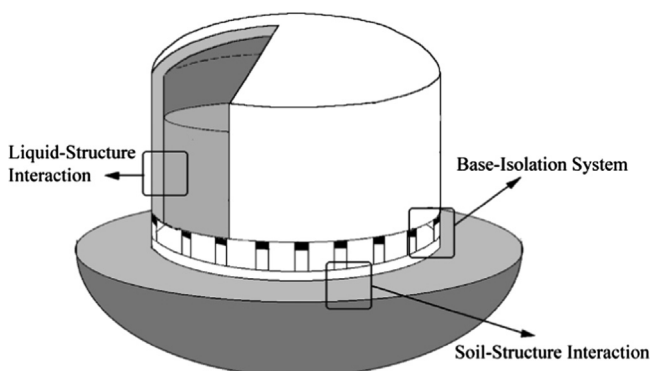


Fig. 2. Sub-systems of the liquid-storage tank with base isolation [40].

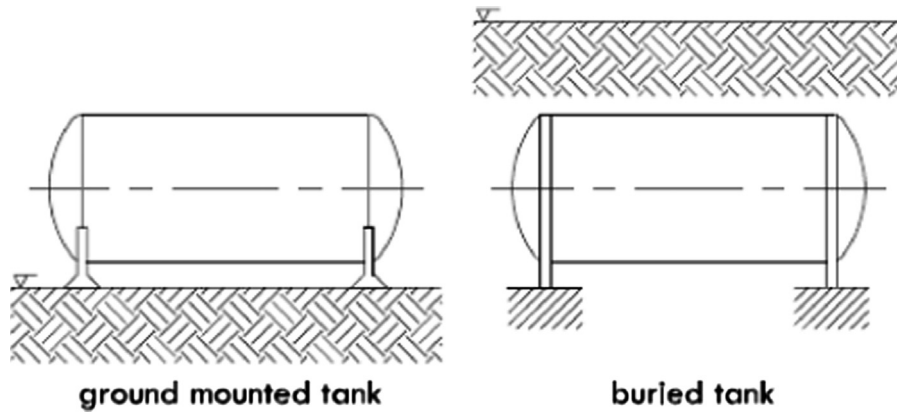


Fig. 3. Ground-mounted and buried horizontal cylindrical tanks supported at the ends [48].

stability of cylindrical shells:

$$D \nabla^8 w + \frac{Et}{R^2} \frac{\partial^4 w}{\partial x^4} + \nabla^4 \left(N_x \frac{\partial^2 w}{\partial x^2} + 2N_{x\phi} \left(\frac{1}{R} \right) \frac{\partial^2 w}{\partial x \partial \phi} + N_\phi \left(\frac{1}{R^2} \right) \frac{\partial^2 w}{\partial \phi^2} \right) = \nabla^4 p \tag{3a}$$

where $w = w(x, \phi)$ is the deflection of the shell at the axial-distance coordinate x and circumferential-angle coordinate ϕ , the stress resultants N_x and N_ϕ are the direct forces per unit length in the x and ϕ directions, respectively, $N_{x\phi}$ is the shear stress resultant with respect to the x and ϕ coordinate directions, R is the radius of the cylinder, E is the Young modulus, t is the shell thickness and D is the flexural rigidity of the shell. The Laplace operator ∇^2 and its powers are given by

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{R^2} \frac{\partial^2}{\partial \phi^2}; \quad \nabla^4 = (\nabla^2)^2; \quad \nabla^8 = (\nabla^4)^2 \tag{3b}$$

An approximate solution of Eq. (3a) was obtained by employing Galerkin’s method of assuming a deflection function $w(x, \phi)$. The results were presented in the form of plots of relative critical shell thickness $(t/R)_{cr}$ versus the relative length of the cylinder, L/R . Although based on elastic analysis with no allowance for imperfections, these plots are useful for the preliminary design of ground-based and buried cylindrical tanks.

In a related study, Magnucki et al. [49] focussed attention on ground-based horizontal cylindrical tanks with ellipsoidal heads, and investigated the optimum length-to-radius ratio (L/R) that minimises the mass of the tank for a given capacity of the tank, taking strength and stability as the criteria for design. Based on analytical parametric studies, they proposed formulae for optimal proportions of ground-based horizontal cylindrical tanks. Such results can be useful in the preliminary stages of design.

A horizontal cylindrical tank with a bulge in the middle (looking like an elongated wine barrel) has recently been considered by Jason and Magnucki [50]. The geometrical parameters of this barrel shape are illustrated in Fig. 4. The barrelled tank is a shell of revolution with a horizontal axis of revolution, and a constant meridional radius of curvature A that is large in comparison with the central (maximum) cross-sectional radius R of the barrel. The double curvature of the shell was expected to enhance the buckling strength of the shell, and to reduce the extent of the bending-disturbance zone adjacent to the end closures.

Using the finite-element method, Jason and Magnucki explored the buckling behaviour of the tank for various loading conditions, and confirmed that the buckling loads for this shell geometry were higher in comparison with those of cylinders of the same average radius. They produced plots of critical buckling loads versus key shell

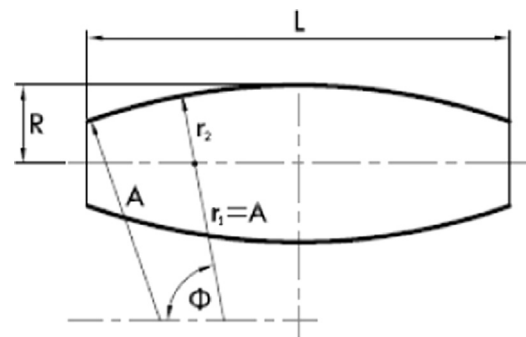


Fig. 4. Geometric parameters of a barrel-shaped horizontal cylindrical tank [50].

geometric parameters, which are useful to anyone wishing to design this type of tank.

5. Studies of conical tanks and related assemblies

The conical tank (and its conical–cylindrical variant) is a popular form of liquid-containment vessel, particularly for elevated water storage. The last 12 years have seen an extension of research efforts commenced over the past 3 decades, involving plain as well as stiffened conical shells. As with cylindrical tanks, buckling problems have dominated the research on conical tanks, on account of their thinness.

Unstiffened conical frusta subjected to uniform external pressure have been experimentally investigated by Golzan and Showkati [51], assuming conditions of full lateral restraint (but free rotation) at the shell edges. They faced the challenge that in manufacturing small-scale laboratory specimens, the imperfections that are produced are relatively large (given the smaller size of the model) in comparison with imperfections in full-scale conical structures, making the extrapolation of findings to full-scale situations somewhat problematic. Nevertheless, these investigators found that the difference between the initial buckling load and the peak buckling load was substantial, demonstrating considerable post-buckling load-carrying capacity, a finding consistent with the earlier work of others [4].

Excessive compressive meridional stresses in the bottom regions of elevated liquid-containing conical tanks are the main cause of buckling in the shell. One would therefore expect the provision of longitudinal local stiffeners in these regions to enhance buckling capacity. El Damatty et al. [52] studied the effect of stiffening the bottom portion of an existing steel conical tank, by longitudinally welding rectangular strips on the outer surface of the shell. The study considered the case when the stiffeners are free at

the bottom edge, and when the stiffeners are built into a concrete base supporting the conical tank. The latter is only possible for new construction, rather than as a strengthening measure.

Based on a parametric study using FEM analysis, they found that the addition of stiffeners free at the base increased the buckling load by 35–64%, while stiffeners embedded in the base gave an increase of 71–136%. In a follow-up study published a year later [53], they developed a design procedure based on these findings, suitable for strengthening existing tanks, and designing new ones.

The configuration of a conical tank with a cylindrical extension at the top is a popular solution for elevated water storage. Hafeez et al. [54] investigated the buckling behaviour of such tanks under hydrostatic pressure, using a finite-element program. The influence of geometric design parameters on the buckling capacity of conical–cylindrical tanks, as well as the effect of geometric imperfections and residual stresses due to welding, were studied. Only circumferential welding (which induces hoop residual stresses) was considered in their study, since they deemed longitudinal residual stresses (due to longitudinal welds) to be non-critical.

The above study found that the buckling capacity of the tanks increased with shell thickness and steel yield strength (as expected), and decreased with increase in height, bottom radius and angle of inclination of the cone with respect to the vertical axis of revolution of the tank. The last parameter was found to have the greatest effect; for instance, an increase in the meridional angle from 30° to 45° reduced the buckling capacity by as much as 65%. The study also found that residual stresses reduced the buckling capacity of the tanks by anything from 9% to 30%.

Sweedan and El Damatty [55] noted that the cause of failure of several conical–cylindrical tanks around the world has been the lack of adequate design guidelines. In an attempt to address this shortcoming, they proposed a simplified design procedure to ensure the safety of such tanks against hydrostatically-induced buckling. A finite element program was used to conduct the numerical investigations. They proposed a conservative design approach for buckling that simply ensured that yielding does not occur at any point on the tank surface, arguing that local yielding usually precedes buckling. They derived a stress magnification factor that relates the maximum overall stresses in the shell (taking into account bending effects, imperfections, etc) to the theoretical membrane stresses calculated from the static equilibrium of the shell under hydrostatic pressure loading. Plots of this factor were given for various geometric parameters of the tank.

As regards the dynamic behaviour of conical tanks, we note that in 2002, Sweedan and El Damatty [56] investigated elevated conical steel vessels on the basis of both experimental model tests and numerical FEM modelling. They considered the case when the conical tanks were empty, to obtain natural frequencies, mode shapes, generalised mass and generalised stiffness parameters. This data was intended for use in dynamic analyses of empty conical tanks subjected to wind loads (during construction or periods of maintenance), and liquid-filled conical tanks subjected to wind or earthquake loads. Through a numerical parametric study, they derived charts that described the variation of fundamental frequency with the height H , bottom radius R and thickness t of the tank, for both open-top tanks and fixed-roof tanks, in the ranges for H of 5–9 m, for R of 3–5 m and for t of 6–28 mm, while the angle of inclination of the cone was kept constant at 45°. They found that the fundamental mode of vibration was governed by $\cos n\theta$ (circumferential harmonics), where n varied between 3 and 9 for open-top tanks, and between 7 and 13 for fixed-roof tanks.

The above study was extended to conical–cylindrical tanks by El Damatty and his co-workers, in a paper published in [57]. The tanks were again assumed to be empty. On the basis of shake-table

testing, numerical FEM modelling and parametric studies, it was found that, as for pure conical tanks, the fundamental mode had a $\cos n\theta$ variation (n varying between 3 and 7) along the circumference. The natural frequencies of the first three modes of vibration had very close values (within 10% of each other). The existence of a number of different modes within a narrow range of frequencies is a characteristic of shell structures in general. Plots of the variation of fundamental frequency with basic geometric parameters were produced.

In a related experimental study, also published in 2005, El Damatty et al. [58] conducted shake-table tests on the small-scale model investigated earlier [49], but this time with the model filled with water. The first two modes of vibration of the liquid-shell system were found to have $\cos n\theta$ ($n > 1$) circumferential variations. These modes produced a localised effect which did not lead to base shear or overturning moment. On the other hand, the $\cos \theta$ modes, which are associated with base shear and overturning moment, only appeared as higher modes of vibration. Sweedan [59] proposed a mechanical model for evaluating the response of liquid-filled conical–cylindrical tanks when subjected to vertical earthquake excitation. Results obtained on the basis of the model agreed reasonably well with results previously published in the literature.

In the past five years, some attention has been paid to the problem of the optimum design of conical–cylindrical tanks. Using a genetic algorithm optimisation technique in combination with nonlinear finite-element modelling, Ansary and co-workers have considered the optimum design of *unstiffened* steel tanks [60], and in a follow-up study [61], the investigation has been extended to *stiffened* tanks. In both cases, the aim has been to select a set of design variables which best satisfy structure safety requirements while achieving minimum weight (hence minimum cost).

6. Investigations of toroidal tanks

Toroidal shells find application as pressure vessels and for holding certain types of liquids (including liquefied gases). Most toroidal vessels are of circular or elliptical cross section. The construction can be in metal, fibre-reinforced plastics or high-strength composites. Special winding techniques are often employed to enhance the structural performance of composite toroidal tanks of the type used for hydrogen storage [62].

Recent theoretical investigations of toroidal vessels have employed a combination of appropriate shell theory and numerical techniques. For instance, Xu and Redekop [63] applied classical shell theory in conjunction with the Differential Quadrature Method (this converts the differential equations of the shell to a set of linear simultaneous equations) to determine natural frequencies of orthotropic toroidal shells of elliptical cross-section.

In a paper published in 2008, Zhan and Redekop [64] studied the mechanics of ovaloid metal toroidal tanks of the type used for holding liquid petroleum gas (LPG). The cross-sections of these tanks were formed by combining a number of different curves, as illustrated in Fig. 5. They carried out a finite-element analysis to determine natural frequencies of vibration, buckling loads and collapse pressures, and conducted a parametric study to establish how these are influenced by shell size, shell thickness and support conditions. Although there were zones of compressive stress in the toroidal tank, the buckling pressures obtained were much higher than the respective collapse pressures (associated with large plastic deformations), suggesting that plastic collapse (rather than buckling resistance) governs the failure of this type of tank. Shell thickness had the greatest influence on buckling resistance, while boundary conditions had a very limited effect.

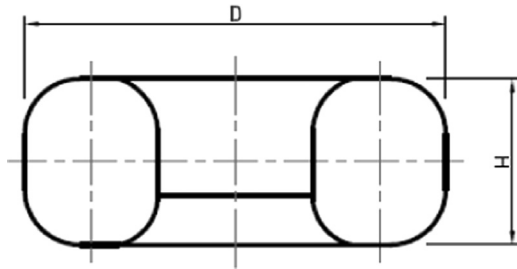


Fig. 5. A toroidal tank for holding liquid petroleum gas [64].

A year later, Zhan and Redekop [65] investigated (using finite-element modelling) two phenomena in a pressurized ovaloid metal toroidal tank: (i) stress concentration around the nozzle, and (ii) displacements due to impact by a flat-nosed projectile. They conducted parametric studies to establish the influence of location of nozzle, or location of impact, on the behaviour of the tank. They found that the stress concentration around nozzles was higher when the nozzle was placed at the intrados rather than at the extrados, and that the impact effects on ovaloid toroidal tanks were basically similar to those for circular toroids of the same cross-sectional dimensions.

Toroidal pressure vessels often need to have relatively thick walls, for which thin-shell theories are not adequate. In a recent study, Wang and Redekop [66] have adopted a shear-deformation shell theory to investigate the free vibration characteristics of moderately-thick and thick toroidal shells. The ensuing equations have been solved by the Differential Quadrature Method. Their formulation gives good results in the range $3 \leq r/t \leq 100$.

7. Junction and other discontinuity problems

Shells of revolution, and associated bending phenomena, have been the subject of numerous investigations in the past. Analytical approaches have been very fruitful in investigating the stresses and deformations in shells of revolution within the elastic range [1,2]. The last 10 years have seen the further development of simplified analytical formulations for calculating shell stresses in the vicinity of geometric discontinuities, shell junctions and ring beams, and for evaluating related effects within ring beams.

The junction problem of the shell of a cylindrical tank, its bottom-plate closure, and supporting concrete ring wall, was considered by Wu and Liu [67] in a paper published in 2000. Compatibility conditions between the shell and the bottom plate were used to calculate the stresses transferred to the plate, and hence to design the plate. The approach used by these investigators was basically analytical (based on the linear elastic theory of plates and shells), and amenable to closed-form mathematical solution. The authors found that the distribution of bending stresses in the bottom circular plate had a rapidly decaying character as one moved inward from the junction with the shell.

Post-tensioned concrete reservoirs may be visualized as assemblies of plate, shell and ring-beam elements. In a paper published in 2002, Oztorun and Utku [68] described a computer program to evaluate the stresses in the reservoir, based on a flexibility formulation of the classical equations of shells, plates and ring beams. The cylindrical tank was assumed to have a dome or flat-slab closure at the top, a flat-slab closure at the bottom, and interposing ring beams at the top and the bottom. The formulation took into account interaction of junction effects at the two ends of the cylinder (short-shell theory). It was intended to provide a practical alternative to FEM analysis, at less cost.

Junction and shell-discontinuity problems in egg-shaped liquid-containment shells of revolution were studied by Zingoni

[69,70], with sludge digesters in mind. Such vessels comprised spherical portions of different radii smoothly joined to form a large tank of closed profile, in which bending disturbances occurred at the junctions on account of the abrupt changes in the meridional radius of curvature of the shell at these locations. A simplification of the Reissner–Meissner equations governing the bending of shells of revolution led to a fourth-order differential equation, allowing closed-form solutions for stresses in the vicinity of the junctions to be obtained.

Based on a simplification of the equations of the axisymmetric bending of conical shells, Zingoni [71] developed analytical solutions for discontinuity stresses around the junctions of double-cone pressure vessels and liquid-filled tanks. These studies showed that the analytical approach can be very useful as a tool for investigating shell stresses in tanks operating under service conditions, and for conducting extensive parametric studies during the preliminary design stages, provided that mathematical solutions of reasonable accuracy can be obtained for the shell geometries in question.

Multi-segmented spherical tanks are the subject of current investigation at the University of Cape Town. These are assemblies of spherical lobes (bulging segments) with a common axis of revolution (Fig. 6). Not only is the whole assembly attractive, but also the lobed profile and vertically elongated configuration has enhanced storage capacity, while the inward-pointing junctions confer additional rigidity (to the shell) against the outward push of the hydrostatic pressure. However, discontinuity effects need to be taken into account, particularly in the lower regions of the tank.

To reduce stress concentrations and other unfavourable discontinuity effects, transition ring beams between the cylindrical and conical parts of a steel tank are often provided in the form of T-beams. Fig. 7 shows a typical transition junction with a T-shaped ring beam. Teng and Chan investigated the elastic buckling strength of such T ring beams in one study [72], and the plastic buckling strength in another study [73]. In the first paper (published in 2000), the authors proposed a simple approximation for the elastic buckling strength of the ring beam in terms of its inner-edge circumferential compressive stress. In the second paper (published in 2001), they developed a procedure for the estimation of plastic out-of-plane buckling strength, and two design approximations were proposed as lower bounds to more exact finite-element analyses.

In a very recent paper [74], Khalili and Showkati have considered the buckling behaviour of T ring beams under internal pressure. Their study was based on experimental investigation and nonlinear finite-element modelling. They came to the conclusion that post-buckling behaviour of T ring beams under internal pressure is stable. Their results have also confirmed the conservative nature of the design proposals of Teng and Chan [73].

8. Non-conventional shell forms for liquid containment

Exploring more efficient shell forms for high-capacity liquid containment seems to have been a neglected area of study in the past 20 years, giving the impression that there is no longer any room for improvement, which is certainly not the case. In the design of containment vessels for sludge digestion at wastewater treatment works, non-conventional options take the form of vertically-elongated shells of revolution of smoothly-varying profiles with rounded ends (giving the appearance of a giant egg), and variants of these with pointed ends (ogival shells) or comprising multi-segmented conical assemblies that approximate the egg-shaped profile. These innovative forms offer several operational advantages over conventional sludge digesters (usually squat cylindrical tanks with flat or domical tops and gently sloping conical floors).

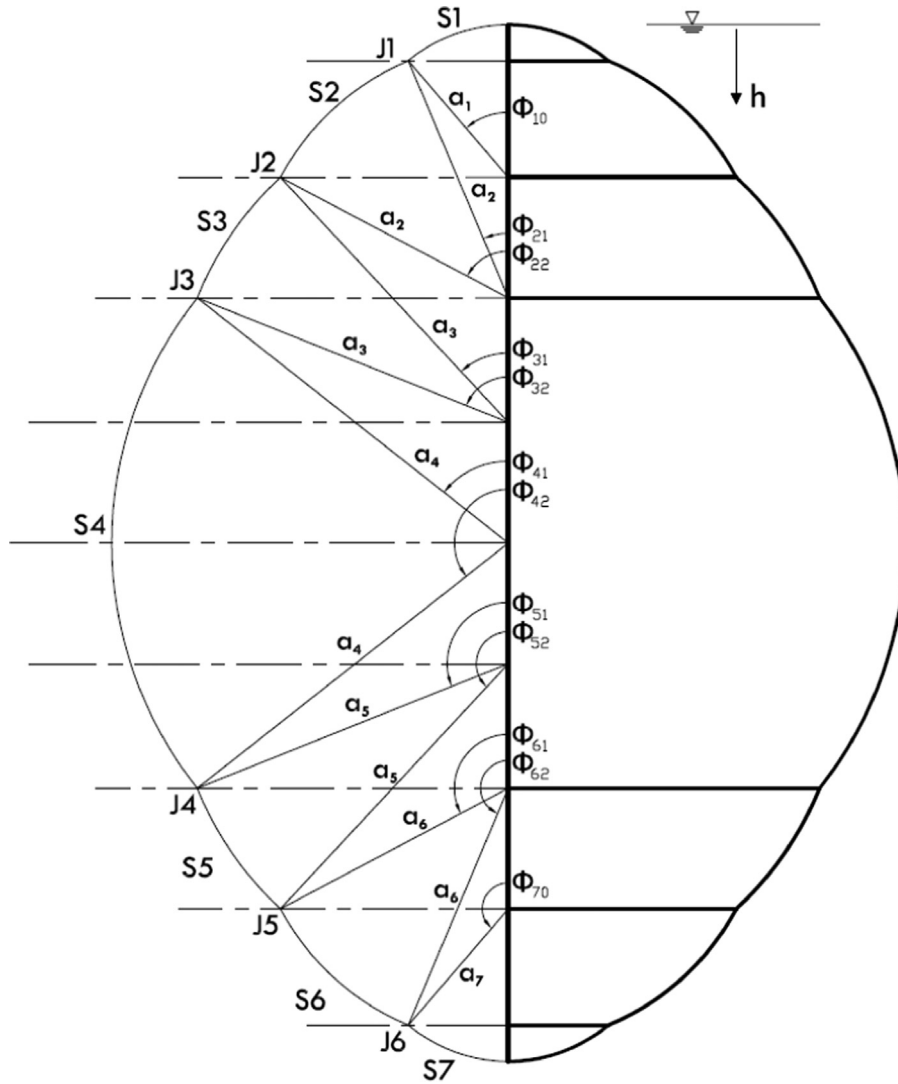


Fig. 6. Multi-segmented spherical tank comprising several shells (S) and junctions (J).

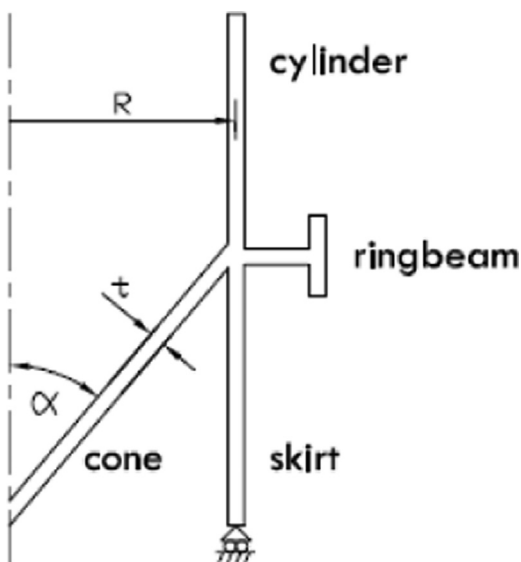


Fig. 7. Typical transition junction with a T-shaped ring beam [72].

The mixing patterns in a conventional digester and an egg-shaped digester are illustrated in Fig. 8. The vertically-elongated smooth profile of the egg shape is conducive to good circulation of the sludge, ensuring that the accumulation of sludge at the bottom of the digester is minimised. The little deposits that do collect at the bottom are easy to remove as they collect in a relatively small area, and the removal of these deposits may be carried out on a continuous basis. The removal of the crust that forms on the surface of the sludge is also facilitated by the tapered shape of the egg shape. Other advantages include lower heat losses (owing to the smaller surface-to-volume ratio of the egg shape relative to that of the cylindrical shape), higher methane-generating capacity (for use in heating the sludge to optimum digester operating temperatures), and a generally more elegant appearance (despite the higher profile of the egg shape). All this adds up to reduced maintenance costs over the long term, despite the higher initial construction costs of the egg shape.

The structural feasibility of non-conventional sludge digesters in the form of thin shells of revolution has been the subject of a programme of research undertaken over the past 12 years at the University of Cape Town. Noting the scarcity of guidelines on the design of such structures, these efforts have centred around

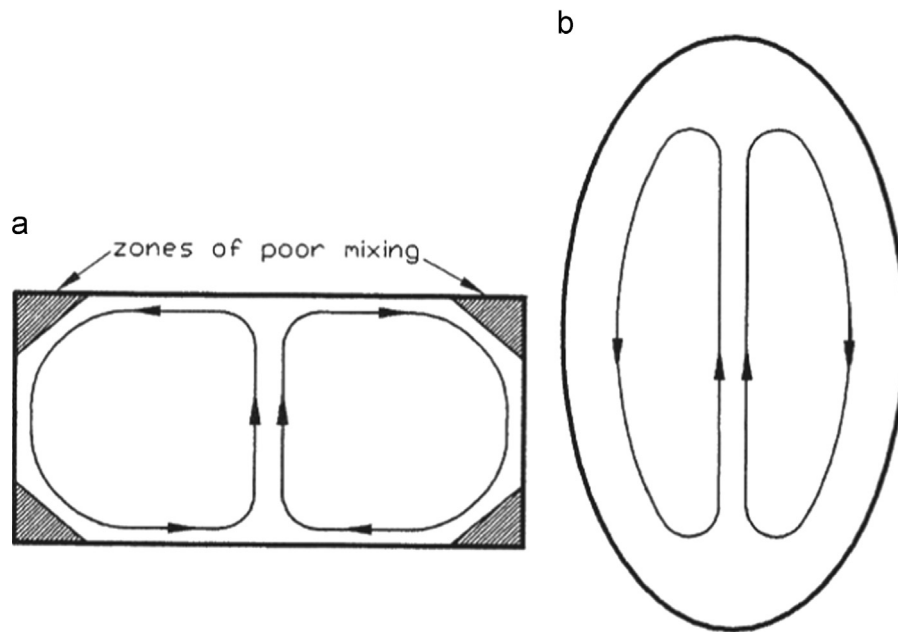


Fig. 8. Typical mixing patterns in sludge digesters: (a) conventional cylindrical digester; (b) egg-shaped digester.

developing usable analytical methods for the shell problems in question, deriving closed-form results for practical use, conducting parametric studies to allow general trends in structural behaviour to be established, and proposing recommendations for design. The studies have initially focussed on the effects of hydrostatic loading. Some of the geometries that have been investigated are depicted in Fig. 9.

At the beginning of the programme, an investigation was undertaken of an egg-like vessel in the form of a shell of revolution consisting of spherical top and bottom closures of radius a and half-angle of opening φ_0 (this angle being typically 45° to 70°), connected by an ogival middle portion of meridional radius of curvature A , as shown in Fig. 9(a). The spherical ends were assumed to meet the middle part tangentially, implying a discontinuity in meridional radius of curvature, but not in slope, at the junctions. Of interest in the study were the membrane stresses generated in the shell as a result of the contained hydrostatic internal pressure, and, in particular, the discontinuity stresses that occur at the junctions of the various regions. The theoretical approach, closed-form analytical results, conclusions and design recommendations have been reported in a two-part article [69,70].

For this configuration, it was observed that junction effects are relatively small in comparison with membrane effects in the lower part of the tank, and the design of the shell will be governed primarily by membrane stresses prevailing in the lower regions. In order to control the steeply increasing membrane meridional compression (which may give rise to buckling problems in the thin shells of the type in question) and membrane hoop tension (which may give rise to cracking problems in the concrete) as one moves towards the bottom of the digester, a number of measures may be taken. The shell may be thickened in the lower half of the digester shell. Tensile reinforcement should also be provided, or prestressing adopted. The support ring of the digester should also not be too low, to ensure that excessive membrane stresses (that tend to occur towards the bottom of the tank) are cut-off.

In another study [75], a parabolic ogival shell was considered. The shape of this is shown in Fig. 9(b). Here the shell of revolution is formed by rotating a parabola that is symmetrical about the horizontal x axis, about the vertical y axis (which therefore is the axis of revolution of the shell). With its bulging middle and

pointed ends, this shape is well-suited for high-capacity sludge containment, and easy removal of both surface crust and bottom deposits. Moreover, the absence of loading and geometric discontinuities over the entire surface of the shell implies a near-membrane state of stress in the entire shell, allowing the study to be conducted solely on the basis of the membrane theory. For this configuration, the stress distribution was expressed in terms of a single governing parameter, $\xi = H/D$ (the height-to-diameter ratio of the vessel), greatly facilitating a parametric study of the problem.

It was found that for parabolic ogival vessels of the same shape (that is, vessels of the same height-to-diameter ratio ξ), stress resultants in the shell are directly proportional to H^2 (or to D^2 , since $D \propto H$). For instance, doubling the height H or diameter D of the tank, while maintaining the parameter ξ constant, quadruples the stress resultants in the shell. This is how the scale of the structure affects its design. The range $1.5 \leq \xi \leq 2.0$ is recommended for practical egg-shaped digesters of parabolic ogival profile, since the slope of the shell is sufficiently steep at the poles ($37^\circ \leq \varphi_0 \leq 45^\circ$) to allow effective prestressing. The overall conclusion was that from both a structural and functional point of view, the parabolic ogival profile is suitable for adoption in the design of egg-shaped sludge digester shells.

Given the costly nature of formwork for concrete shells with continuously-varying slope (in order to achieve the smoothly-curved “egg” profile), the possibility of adopting a series of straight segments for the shell meridian (which are easier and cheaper to construct), while preserving the vertically elongated and good sludge-mixing properties of the basic egg shape, was also considered. A double-cone or rhombic configuration (Fig. 9(c)) is the simplest of such an assembly, with the compound cone-frusta variant (Fig. 9(d)) providing a better solution. Theoretical results for arbitrary cone-cone assemblies [71] were applied to study the structural feasibility of these digester options.

For typical sizes of tanks likely to be encountered in practice, it was found that discontinuity stresses around the equatorial junction of the arrangement in Fig. 9(c) are relatively large in comparison with membrane stresses. The sharp discontinuity stresses there necessitate the placement of a ring beam at the junction, or the reduction of the slope discontinuity through the

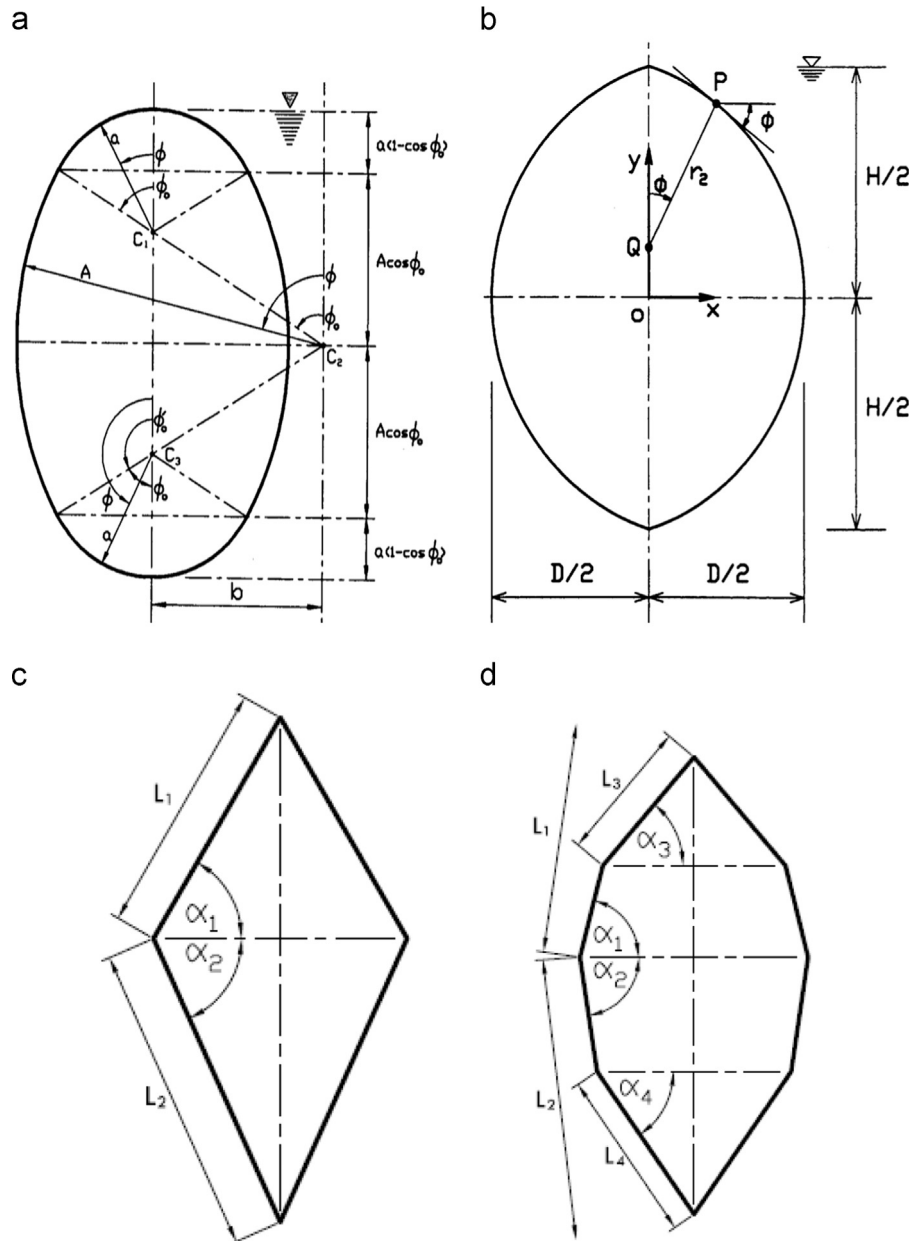


Fig. 9. Non-conventional forms for sludge digesters: (a) spherical ogival shell; (b) parabolic ogival shell; (c) double-cone assembly; (d) multi-cone assembly.

adoption of the variant in Fig. 9(d), which has the added benefit of enhancing the containment capacity of the digester for the same overall proportions (height and diameter) of the digester.

9. Concluding remarks

Covering the period 2000 up to now, this review has covered research on the strength, stability, vibration and dynamic behaviour of liquid-containment shell structures, and discussed how this research has influenced design practice. Recent findings on the feasibility of new shell forms for liquid containment have also been briefly discussed.

The survey has revealed that research on containment metal shells (typically steel tanks and silos) continues to be dominated by studies of cylindrical vessels and, albeit to a lesser extent, conical vessels. These geometries are the commonest in practice owing to the ease of fabrication of shells of single curvature. Much is now understood about the buckling behaviour of circular

cylindrical tanks under external wind pressure, and their dynamic response under wind and seismic excitation. This is reflected in the content of existing design standards on containment shells [8,30,32].

Relatively few studies have been reported on liquid-storage metal shells of double curvature, which include spherical, ellipsoidal and toroidal tanks. These forms have found widespread application in pressure-vessel technology, but it appears their application for high-capacity liquid containment is hampered not only by their higher costs of fabrication, but also by the lack of comprehensive design guidelines within existing codes of practice. It is evident that part of the research on metal shells of double curvature should focus on the development of more cost-effective methods of fabrication, while another part should aim at providing the engineer with sound design guidelines.

The specific geometry of shells of double curvature (ellipsoidal, paraboloidal, hyperbolic, toroidal etc) has a major influence not only on the stress distribution in the shell, but also on the dynamic characteristics, buckling resistance and post-buckling behaviour of

the shell. Would metal tanks in the form of ellipsoids of revolution exhibit significantly higher buckling resistance to external wind pressure than spherical tanks of the same capacity? From a buckling point of view, what is the best cross-sectional shape for a toroidal vessel in resisting hydrodynamic or seismic loads? These are some of the questions which new research needs to address. Once sufficient understanding of the behaviour of these alternative shell forms is available, one hopes that suitable guidance would be incorporated into current standards on shell design [8,30,32].

Elevated concrete water tanks can be constructed in a great variety of shapes owing to the mouldability of concrete into any desired shape, but surprisingly not much exploration of new shell forms for concrete tanks has been undertaken to date. Exploration of new and more efficient shell configurations holds much promise in seeking solutions for high-capacity elevated liquid storage (super-sized tanks). New research is required to address problems associated with shell behaviour on this scale. Related problems involve submarine concrete shell structures for offshore installations; these have to resist external hydrostatic and hydrodynamic loads. Again, the response of the structure very much depends on its shape.

An observation concerns the tools that are currently being employed for research in this area. Numerical methods (such as the Finite Element Method, the Finite Difference Method, the Boundary Element Method, and the Differential Quadrature Method) have permitted solutions to be found for problems that were previously intractable, and to explore complex dynamic and buckling behaviour of shells. The pursuit of rigorous mathematical solutions seems to have dampened, yet there are tremendous rewards (including a deeper understanding of phenomena) that can be gained by a more fundamental approach, without necessarily abandoning the powerful tools of numerical methods. Of course, experimental methods remain indispensable, despite the high costs often associated with testing.

The significant discrepancies between experimental and numerical results noted in many of the reported studies point to a need for better theories and further development of existing numerical techniques, particularly in simulating complex phenomena like buckling behaviour under fire conditions (oil storage tanks). Further effort needs to be invested in the development of sharper research tools, for a better understanding of complex shell phenomena.

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